SHAPE MATCHING OF TWO-DIMENSIONAL OCCLUDED OBJECTS

Bir Bhanu

Ford Aerospace and Communications Corporation
Newport Beach, California 92660, USA

Abstract

A hierarchical stochastic labeling technique to do shape matching of 2-D occluded objects is presented. The technique explicitly maximizes a criterion function based on the ambiguity and inconsistency of classification. The hierarchical nature of the algorithm reduces the computation time and uses results obtained at low levels to speed up and improve the accuracy of results at higher levels. The 2-D shapes are represented by their polygonal approximation. For each of the objects participating in the occlusion, there is a hierarchical process. These processes are executed in parallel and are coordinated in such a way that the same segment of the apparent object, formed as a result of occlusion of two or more actual objects, is not matched to the segments of different actual objects. This problem is solved by combining the gradient projection method and penalty function approach. Objects participating in the occlusion may move, rotate, undergo significant changes in the shape and their scale may also change. Results are presented when two or three objects partially occlude.

I. INTRODUCTION

Matching of occluded objects is one of the prime capabilities of any shape analysis system. We view the occlusion problem in 2-D basically a boundary matching problem. As compared to the previous studies, the framework presented here provides a mathematical basis for the solution of the occlusion problem [1].

II. PROBLEM FORMULATION

Consider a general case in which M ($\geq$ 2) actual objects, called models $(X_1,\ldots,X_M)$ occlude one another to form a single apparent object called the object. Fig. 1 shows the block diagram of the algorithm when two models occlude each other. Let a model X_m $(m = 1,\ldots M)$ be represented by $X_m = (T_1,T_2,\ldots,T_{N_m})$, where N_m is the number of segments in the polygonal path representation. Similarly, let $O = (O_1,O_2,\ldots,O_{L-1})$ be the polygonal path representation of the object. The object has L-1 segments. We want to match the segments of the models agains segments of the object such that the following two conditions are satisfied.

1) None of the segments of the different models are assigned to the same segment of the object. This is called the occlusion condition. It is necessary for the labeling to be unambiguous.

2) Those segments of the models which do not match to any of the segments of the object are assigned to the nil class, i.e., no match class.

We are trying to identify parts of the models within the object. We designate the object segments as classes, and the model segments as units. Let the nil class be denoted by O_L. To each of the units T_i of a model X_m, we assign a probability denoted by $p_{im}(k)$ to belong to class O_k. This is conveniently represented as a vector $\vec{p}_{im} = [p_{im}(1),\ldots,p_{im}(L)]^T$. The set of all vectors $\vec{p}_{im}$ $(i = 1,\ldots,N)$ is called a stochastic labeling of the set of units. The global criterion that measures the consistency and ambiguity of the labeling over the set of units of a model X_m is given by [1,2],

$$J_m^{(n)} = \sum_{i=1}^{N_m} \vec{p}_{im} \cdot \vec{q}_{im}^{(n)} , \quad n = 1,2 \tag{1}$$

where

$$q_{im}^{(n)}(k) = \frac{Q_{im}^{(n)}(k)}{\sum_{\ell=1}^{L} Q_{im}^{(n)}(\ell)} , \quad \begin{array}{l} n = 1,2 \\ i = 1,\ldots,N_m \\ k = 1,\ldots,L \end{array} \tag{2}$$

and,

$$Q_{ij,m}(k) = \sum_{\ell=1}^{L} C_1(i,k,j,\ell)p_{jm}(\ell), \quad \begin{array}{l} j = i-1,i+1 \\ i = 1,\ldots,N_m \\ k = 1,\ldots,L \end{array} \tag{3}$$

$$Q_{im}^{(1)}(k) = \tfrac{1}{2}(Q_{i\ i-1,m}(k) + Q_{i\ i+1,m}(k)) \tag{4}$$

$$Q_{im}^{(2)}(k) = \sum_{\ell_1,\ell_2=1}^{L} C_2(i,k,i-1,\ell_1,i+1,\ell_2)p_{i-1\ m}(\ell_1)\, p_{i+1\ m}(\ell_2) \tag{5}$$

n denotes the first or second stage of the hierarchy. C_1 and C_2 comapre the local structures of X_m and O at the first and second stage. They are defined as functions of $S_2 x O^2$ and $S_3 x O^3$ into [0,1] where, S_2 and S_3 are two subsets of X_m^2 and X_m^3 defined by,

$$S_2 = \{(T_i,T_j)\},\ i = 1,\ldots,N_m\ \ j = i-1 \text{ or } i+1$$

$$S_3 = \{(T_i,T_{i-1},T_{i+1})\},\ i = 1,\ldots,N_m$$

Let $\vec{v}_m$ be the vector of R^P, $(P = N_m L)$ equal to $(\vec{p}_{1m}, \vec{p}_{2m},\ldots,\vec{p}_{N_m})$. Then the total criterion of consistency and ambiguity for all the M models is,

$$F(\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_M) = \sum_{m=1}^{M} \sum_{i=1}^{N_m} J_{im}^{(n)}(\vec{v}_m), \quad n = 1,2 \tag{6}$$

where $J_{im}^{(n)}(\vec{v}_m) = \vec{p}_{im} \cdot \vec{q}_{im}^{(n)}$. The occlusion condition is

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$$G(\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_M) = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} g(\vec{s}_i,\vec{s}_j) = 0 \quad (7)$$

where $\vec{s}_\ell$ is obtained from $\vec{v}_\ell$ with the elements corresponding to the nil class set equal to zero for all the units of the model X_ℓ and $g(\vec{s}_i,\vec{s}_j)$ is the inner product of the vectors $\vec{s}_i$ and $\vec{s}_j$. Now the occlusion problem can be stated as follows.

Problem Statement (A): Given an initial labeling $\vec{v}_1^{(0)}, \vec{v}_2^{(0)},\ldots,\vec{v}_M^{(0)}$ for the set of M models $(X_1,X_2,\ldots,X_M)$ to belong to various segments of the object, find the labeling $\vec{u}_1, \vec{u}_2,\ldots,\vec{u}_M$ that corresponds to the local maximum of the criterion (6) which is closest to $\vec{v}_1^{(0)}, \vec{v}_2^{(0)},\ldots,\vec{v}_M^{(0)}$ subject to the constraints: (a) If $\vec{u}_m = (\vec{p}_{1m},\vec{p}_{2m},\ldots,\vec{p}_{N_m m})$ then $\vec{p}_{\ell m}$ is a probability vector for $\ell = 1,2,\ldots,N_m$ and $m = 1,2,\ldots,M$. (b) $G(\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_M)$ as defined by (7) be equal to zero.

Note that the criterion (6) is nonlinear. Constraint (a) involves linear equality and nonnegativity restriction, and constraint (b) is nonlinear. In order to solve this optimization problem we use the gradient projection method and penalty function concept [3].

III. OCCLUSION ALGORITHM

In order to solve the problem (A) we define the penalized objective function as,

$$\psi_c(\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_M) = F(\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_M) + \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} d_{ij}\phi_{ij}[g(\vec{s}_i,\vec{s}_j)] \quad (8)$$

where ϕ_{ij} is a penalty function and $\{d_{ij}\}$ are penalty constants. The penalty function is taken as the simple quadratic loss function.

The problem (A) is equivalent to that of maximizing (8) subject to the constraints (a). It is solved by using the gradient projection method applied to the linear constraints. The maximization of (8) subject to the constraints (a) is equivalent to maximizing

$$\begin{cases} \max\limits_{\vec{v}_1} \; F(\vec{v}_1) + S(\vec{v}_1,\ldots,\vec{v}_M) \\ \max\limits_{\vec{v}_2} \; F(\vec{v}_2) + S(\vec{v}_1,\ldots,\vec{v}_M) \\ \vdots \\ \max\limits_{\vec{v}_M} \; F(\vec{v}_M) + S(\vec{v}_1,\ldots,\vec{v}_M) \end{cases} \quad (9)$$

where $S(\vec{v}_1,\ldots,\vec{v}_m)$ corresponds to the second term of (8). In general to solve (9) by maximizing with respect to $\vec{v}_i$ the algorithm can be stated as follows:

1. Pick an initial estimate of $(\vec{v}_1^{(0)}, \vec{v}_2^{(0)},\ldots,\vec{v}_M^{(0)})$. This is the initial assignment of probabilities to the units of the models.
2. Pick the penalty constant $\{d_{ij}\}$ so that it provides a suitable balance between the associated first and second terms of (9). This is done in an automatic manner [1].
3. Determine the maximum $\vec{v}_m^{(n+1)}$ $(m = 1,2,\ldots,M)$ of the unconstrained penalized objective function (9) subject to the constraints (a) by using the value at the present iteration $\vec{v}_m^{(n)}$ and gradient projection method.
4. Pick new penalty constants $\{d_{ij}\}$ in order to rebalance the magnitude of the penalty terms; replace n by n+1 and return to 3.

Under the assumption of the continuity of function F (6) and constraints (7) inherent in (8), the sequence of maxima $\{\vec{v}_m^{(n+1)}\}$ for $m = 1,\ldots,M$ generated by the above algorithm approaches a constrained maximum of the problem defined in (A). Since we are seeking only local maximum, ill-conditioning problems do not occur [3].

IV. EXAMPLES

Example 1: Figure 2 presents a synthetic example, where three models X_1, X_2 and X_3 occlude one another to form an apparent object. We want to identify each of the models within the apparent object. The problem is a kind of "jig-saw puzzle". Table 1 shows the results of labeling All the labels of all the units of X_1, X_2 and X_3 are correct except the label of the unit 5 of the model X_1. This is because of the high similarity of the local structure of the incorrect match.

Example 2: Figure 3 shows gray scale images of industrial parts (fig. 3(b) and (c)) which occlude each other to form an occluded object shown in fig. 3(a). The images shown in fig. 3 are of size 512x512, 8 bits. The images in figs. 3(b) and (c) are reduced by 16 times and the image in fig. 3(c) by 18 times. The reduced images are thresholded and their polygonal approximation is shown in fig. 4. Only the rotation and scale invariant features are used in the initial probability assignment. Label 25 is the nil class. The results are shown in Table 2. Note that all the key assignments of the units are correct.

V. CONCLUSIONS

The technique is found to be very effective when applied to synthetic, industrial and biological images [1]. The computation time varies linearly with the number of objects occluding one another and it took about 1 to 10 minutes on a PDP-10.

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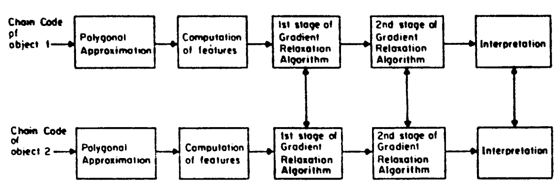


Fig. 1 Block diagram of the occlusion algorithm for the shape matching of two occluding models using two stages of the coordinated hierarchical stochastic labeling technique.

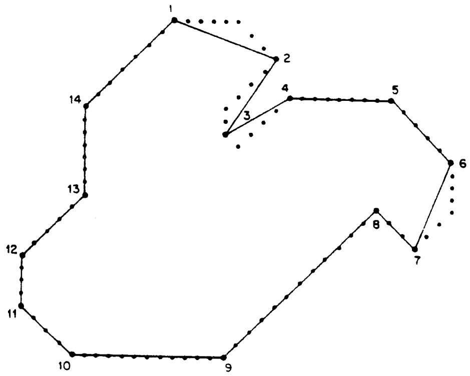


Fig. 2(a) Apparent Object

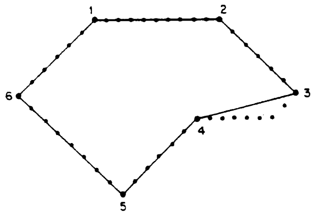


Fig. 2(b) Model X_1

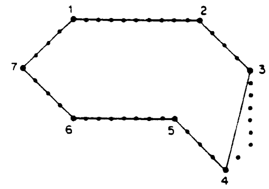


Fig. 2(c) Model X_2

Table 1. Results of Labeling for the Model X_1, X_2, and X_3, Example 1

Labels at Different Iterations

Units of Model X_1, X_2, and X_3	Model X_1 First Stage		Model X_1 Second Stage		Model X_2 First Stage		Model X_2 Second Stage		Model X_3 First Stage		Model X_3 Second Stage	
	0	3	1	6	0	3	1	6	0	3	1	6
1	4	15	15	15	4	4	4	4	4	15	15	15
2	1	1	1	1	5	5	5	5	5	15	15	15
3	15	15	15	15	6	6	6	6	13	15	15	15
4	15	15	15	15	15	7	7	15	1	15	15	15
5	15	15	15	7	13	13	13	15	2	15	15	15
6	15	15	15	15	15	15	15	15	9	9	9	9
7					15	15	15	3	10	10	10	10
8									11	11	11	11
9									14	15	15	12
First Term	-	.97	1.07	2.4	-	1.57	1.38	3.45	-	3.03	3.07	3.69
Penalty Term	-	.29	.32	0	-	.47	.41	0	-	.90	.92	0
Criterion	-	.68	.75	2.4	-	1.09	.97	3.45	-	2.12	2.15	3.69
Penalty Const.	-	.042	.075	-	-	.07	.09	-	-	.13	.21	-

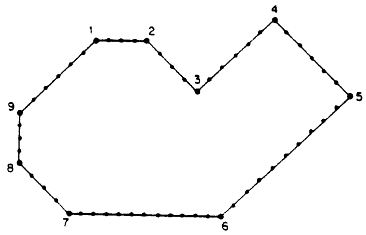
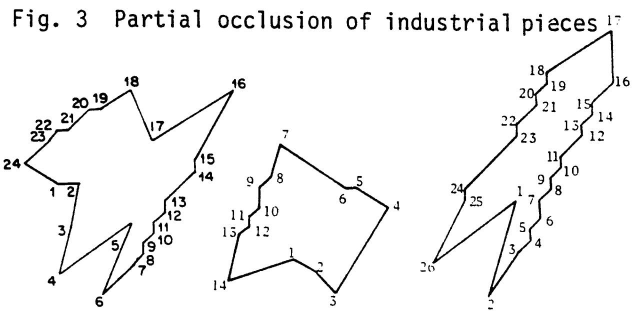


Fig. 2(d) Model X_3

(a) (b) (c)

Fig. 3 Partial occlusion of industrial pieces



(a) (b) (c)

Fig. 4
Apparent Object Model X_2 Model X_1

Table 2. Results of Labeling for the Model X_1 and X_2, Example 2

Labels at Different Iterations

Units of Model X_1&X_2	Model X_1 First Stage		Model X_1 Second Stage		Model X_2 First Stage		Model X_2 Second Stage	
	0	3	1	5	0	3	1	5
1	5	5	5	5	25	25	25	25
2	6	6	6	6	25	25	25	23
3	25	7	25	7	25	25	25	25
4	25	8	8	8	25	25	25	25
5	25	25	25	25	25	25	25	25
6	25	25	25	25	25	25	25	15
7	25	25	25	25	25	18	18	18
8	25	25	25	25	25	25	25	25
9	25	25	25	25	25	25	25	25
10	25	25	25	25	25	25	25	25
11	25	25	25	25	25	25	25	25
12	25	25	25	25	25	25	25	19
13	25	25	25	25	25	25	25	22
14	25	25	25	25	25	25	25	25
15	25	25	25	25				
16	25	25	25	14				
17	25	25	25	25				
18	25	25	25	25				
19	25	25	25	25				
20	25	25	25	25				
21	25	25	25	25				
22	25	25	25	25				
23	25	25	25	25				
24	14	14	14	14				
25	25	25	25	25				
26	25	4	25	25				
First Term	—	4.7	16.2	17.2	--	2.8	5.6	7.1
Penalty Term	--	.47	1.6	0	--	.28	.56	0
Criterion	--	4.2	14.6	17.2	--	2.5	5.1	7.1
Penalty Const.	--	.008	1.7	--	--	.005	.59	--

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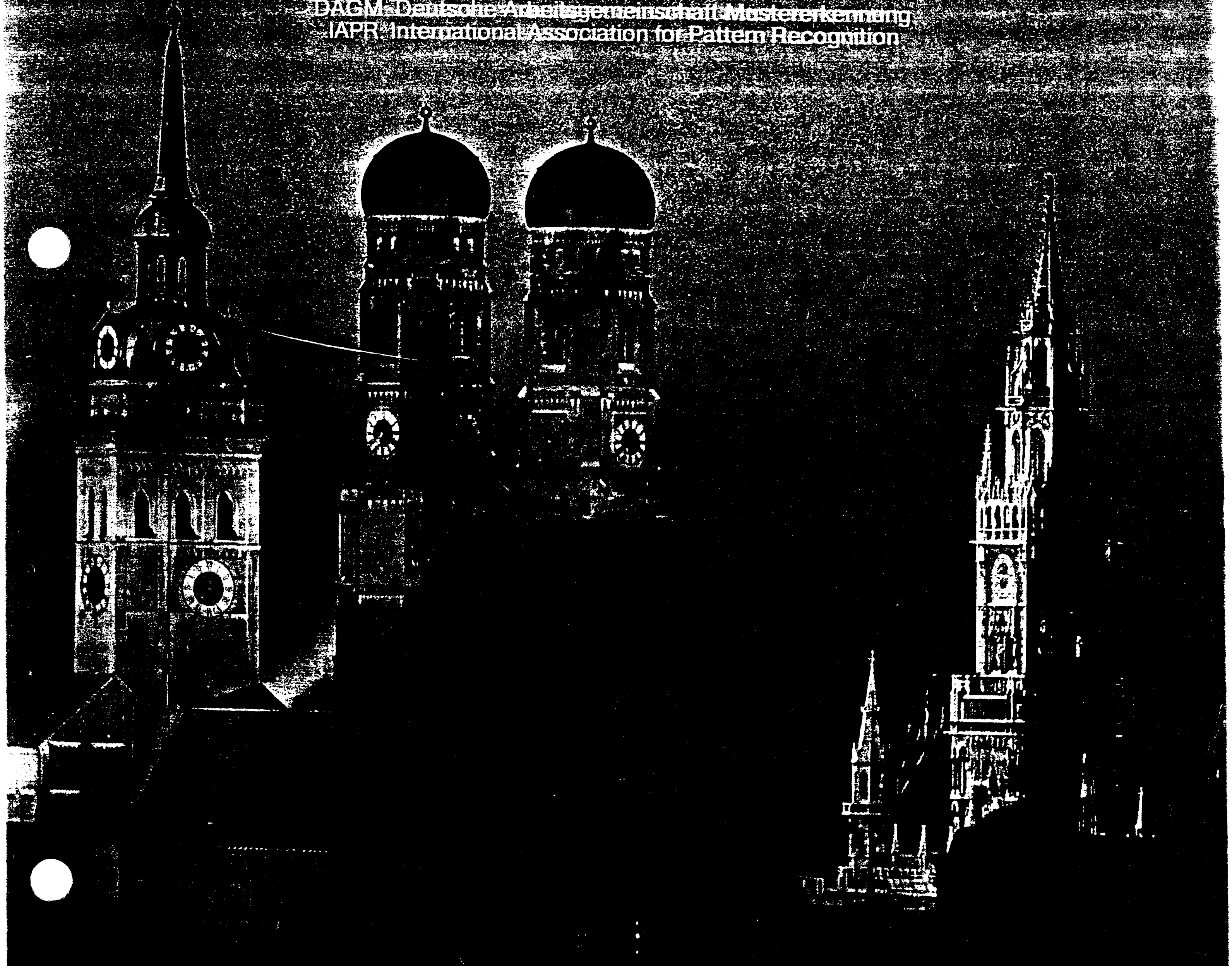

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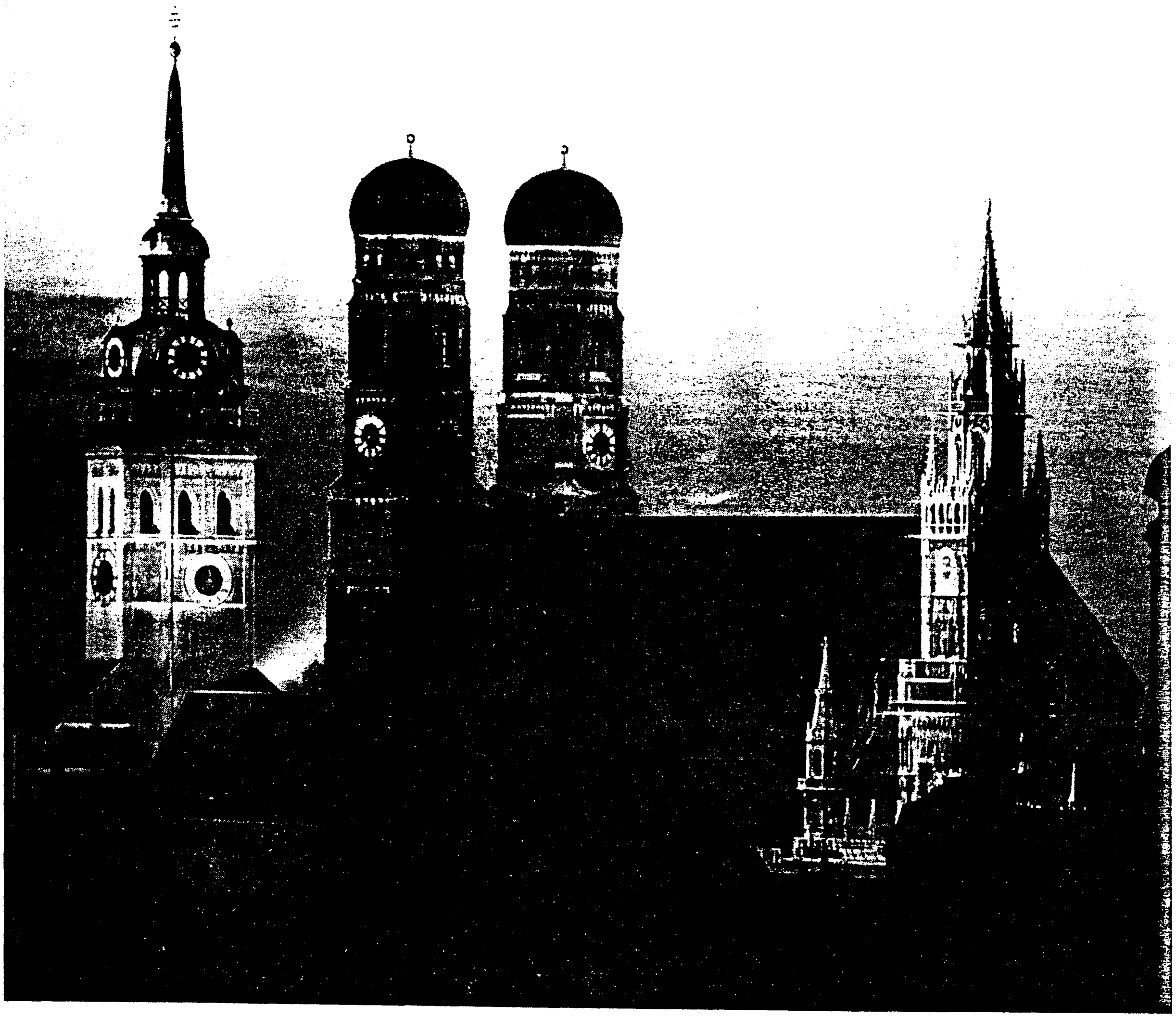

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