SEGMENTATION OF IMAGES USING A GRADIENT RELAXATION TECHNIQUE

Bir Bhanu

Ford Aerospace and Communications Corporation
Newport Beach, CA 92660, USA

Abstract

A gradient relaxation method based on maximizing a criterion function is presented for the purpose of segmentation of images having almost unimodal distributions. The method provides control over the relaxation process by choosing three parameters which can be tuned to obtain the desired segmentation results. Examples are given on several different types of scenes.

I. INTRODUCTION

In this paper we present a gradient relaxation method based on maximizing a criterion function for the purpose of segmentation of images having unimodal gray level distributions. Unimodal distributions are obtained when the image consists mostly of a large background area with other small but significant regions. This is true for most biological and aerial images. As an example Fig. 1 shows a cancer cell image. Our objective is to get the boundaries of all the cells. Gray level histogram of the image is shown in fig. 2. Note that the histogram is almost unimodal and as a consequence there is no reliable way of automatically choosing a threshold for segmenting this image.

II. SEGMENTATION USING GRADIENT RELAXATION

We consider a criterion which is based upon the explicit use of consistency and ambiguity to define a global criterion upon the set of pixels [1,2]. The criterion is the inner product of probability vector $\vec{p}_i$ and consistency vector $\vec{q}_i$. N is the number of pixels in the image, $\vec{q}_i$ is a function of $\vec{p}_i$'s as discussed below and we define the criterion as,

$$C(\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_N) = \sum_{i=1}^{N} \vec{p}_i \cdot \vec{q}_i \tag{1}$$

and carry out its maximization using the gradient projection approach.

Suppose the set of N pixels i= 1,2,...,N fall into two classes λ_1 and λ_2 corresponding to the white (gray value = 255) and black (gray value = 0). The relaxation process is specified by choosing a model of interaction between pixels. We attach to every pixel i the set V_i of its 8 nearest neighbors. Assuming that objects of interest in the picture are continuous we will make like reinforce like and define a compatibility function c such that:

$$\begin{aligned} c(i,\lambda_k,j,\lambda_\ell) &= 0, \quad k\neq\ell, \quad j \text{ in } V_i \text{ for all } i \\ c(i,\lambda_k,j,\lambda_k) &= 1, \quad k=1,2 \quad j \text{ in } V_i \text{ for all } i \end{aligned} \tag{2}$$

The consistency vector $\vec{q}_i$ is then defined as

$$q_i(\lambda_k) = \frac{1}{8}\sum_{j\in V_i}\sum_{\ell=1}^{2} c(i,\lambda_k,j,\lambda_\ell)\, p_j(\lambda_\ell), \qquad k = 1,2,\; i = 1,\ldots,N \tag{3}$$

The maximization of the global criterion (1) means that we are seeking a local maximum close to the initial labeling $\vec{p}_i(0)$ (i = 1,...,N) subject to the constraints that $\vec{p}_i$'s are probability vectors. The maximization of (1) results in a reduced inconsistency and ambiguity. Inconsistency is defined as the error between $\vec{p}_i$ and $\vec{q}_i$. Intuitively this means the discrepancy between what every pixel "thinks" about its own labeling ($\vec{p}_i$) and what its neighbors "think" about it ($\vec{q}_i$). Ambiguity is measured by the quadratic entropy and results from the fact that initial labeling $\vec{p}_i(0)$ is ambiguous ($\vec{p}_i(0)$ are not unit vectors). We are trying to align the vectors $\vec{p}_i$ and $\vec{q}_i$ while turning them into unit vectors. It can be easily seen that each term $\vec{p}_i \cdot \vec{q}_i$ is maximum for $\vec{p}_i=\vec{q}_i$ (maximum consistency) and $\vec{p}_i=\vec{q}_i=$ unit vector (maximum unambiguity).

From the intensity distribution the initial assignment of probabilities is obtained by [2],

$$p_i(\lambda_1) = FACT*\left(\frac{I(i)-\bar{I}}{255}\right)+0.5 \tag{4}$$

where $\bar{I}$ is the mean of the image, I(i) is the intensity at the ith pixel, and FACT is a function of the intensity. When $I(i)<\bar{I}$, FACT is usually taken between 0.5 and 1 and equal to 1 when $I(i)>\bar{I}$. If the first term of (4) happens to be greater than 0.5 or less than -0.5, then a probability of one or zero respectively is assigned to that pixel.

The gradient of the criterion C in (1) is,

$$\frac{\partial C}{\partial p_i(\lambda_1)} = 2q_i(\lambda_1) \qquad \frac{\partial C}{\partial p_i(\lambda_2)} = 2q_i(\lambda_2) \tag{5}$$

Computing the projection of the gradient, the iteration of $\vec{p}_i$'s is given by [2],

$$p_i^{(n+1)}(\lambda_1) = p_i^{(n)}(\lambda_1) + \rho^{(n)}\left[2q_i(\lambda_1) - 1\right] \tag{6}$$

$$p_i^{(n+1)}(\lambda_2) = p_i^{(n)}(\lambda_2) + \rho^{(n)}\left[1-2q_i(\lambda_1)\right] \tag{7}$$

where $\rho^{(n)}$ is a positive step size. Normally, $\rho^{(n)}$ is kept constant for all pixels during each iteration and is determined to have the largest

CH1801-0/82/0000/0915 $00.75 © 1982 IEEE

possible value such that $\vec{p}_i$'s at the n+1st iteration still lie in the bounded convex region of 2N dimensional Euclidean space defined by $p_i(\lambda_1)+p_i(\lambda_2)=1$ and $p_i(\lambda_k)\geq 0$, k=1,2 and i=1,...,N. However, in the 2 class case considered it is easier to compute a $\rho_i^{(n)}$ for each pixel. This leads to a faster convergence rate. The maximum possible value for $\rho_i^{(n)}$ is obtained from (6) by setting $p_i^{(n+1)}(\lambda_1)=1$ when $2q_i(\lambda_1)-1 > 0$ and $p_i^{(n+1)}(\lambda_1) = 0$ when $2q_i(\lambda_1)-1 < 0$. Thus,

$$\rho_{iMax}^{(n)} = \begin{cases} \left(\dfrac{1-p_i^{(n)}(\lambda_1)}{2q_i(\lambda_1)-1}\right), \text{ if } 2q_i(\lambda_1)-1>0 & (8) \\ \left(\dfrac{p_i^{(n)}(\lambda_1)}{1-2q_i(\lambda_1)}\right), \text{ if } 2q_i(\lambda_1)-1<0 & (9) \end{cases}$$

Since we want to be able to control the rate of convergence and the number of pixels within each class we actually took

$$p_i^{(n+1)}(\lambda_k) = p_i^{(n)}(\lambda_k) + \rho_i^{(n)}\left[2q_i(\lambda_k)-1\right] \qquad (10)$$

and,

$$\rho_i^{(n)} = \begin{cases} \alpha_1\, \rho_{iMax}^{(n)}, \text{ if } 2q_i(\lambda_1)-1>0 \\ \alpha_2\, \rho_{iMax}^{(n)}, \text{ if } 2q_i(\lambda_1)-1<0 \end{cases} \qquad (11)$$

where, k=1,2 and α_1 and α_2 are constants less than unity. A side effect of computing $\rho_i^{(n)}$ for every pixel is that we may not be following the gradient exactly. However, it can be expected that we are approximately in the direction of the gradient and the criterion (1) is still maximized. It is our conjecture that for the two class case, the criterion will always increase. Figure 3 shows the behavior of criterion as the iteration number increases for the cell image when $\alpha_1 = \alpha_2$. The values of α_1 and α_2 can be used to bias a class and control the speed of convergence, hence the control over the relaxation process.

Fig. 4 shows how changing the ratio of α_1 and α_2 allows the control of where we converge by biasing one class. From fig. 3 it can be seen that for a fixed ratio of α_1 and α_2, increasing both of them by a constant factor increases the speed of convergence and fig. 5 verifies this fact that indeed we converge towards a similar result. What we seem to be loosing when increasing α_1 and α_2 is some amount of smoothing: there are a few more isolated small black blobs in fig. 5(b) than in fig. 5(a). The magnitude of α_1 and α_2 controls the degree of smoothing at each iteration and their ratio the bias. The value of FACT does not affect the speed of convergence very much, but it affects the segmentation results [2].

Fig. 6 to 9 show the results of gradient relaxation method at various iterations and corresponding histograms for 4 aerial images. Observe that at the first iteration itself we get two peaks separated by a valley which can be used to automatically select the threshold to obtain segmentation. As the number of iteration increases the two peaks get apart, contrast increases, and the convergence of probabilities takes place as expected. When the peaks are far apart thresholding can be done at the mean value.

III. CONCLUSIONS

The method provides the control over the relaxation process by choosing the α_1, α_2 and FACT parametes which can be tuned to obtain the desired segmentation. The method has been compared with the nonlinear relaxation method in detail in [2] on the basis of segmentation results, convergence rate and the ease with which the relaxation process is controlled.

REFERENCES

[1] M. Berthod and O.D. Faugeras, "Using Context in the Global Recognition of a set of objects: An Optimization Approach," Proc. 8th World Computer Congress (IFIP 80), Tokyo.

[2] B. Bhanu, "Shape Matching and Image Segmentation Using Stochastic Labeling," USCIPI Report 1030, Image Proc. Inst., USC, Los Angeles, Ca., August 1981.

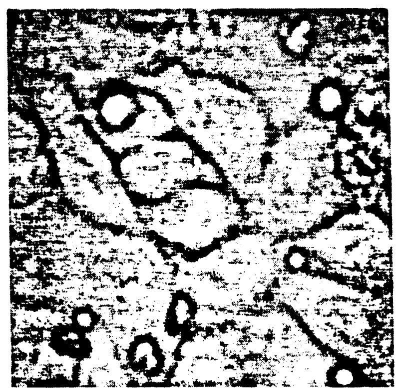


Fig. 1 Cancer Cell Image

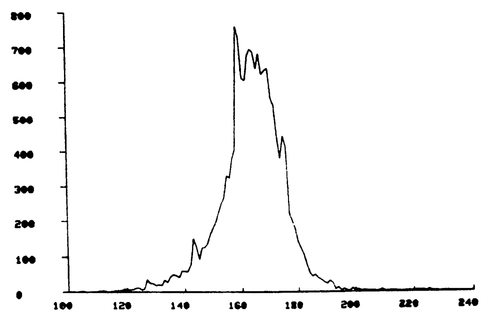


Fig. 2 Histogram of the Image in Fig. 1

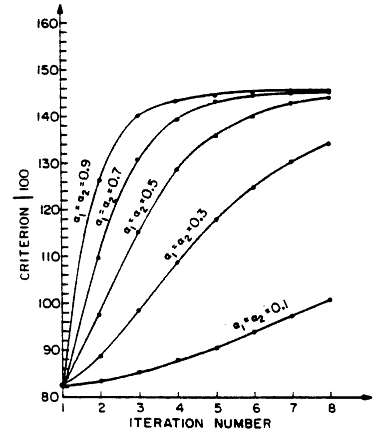
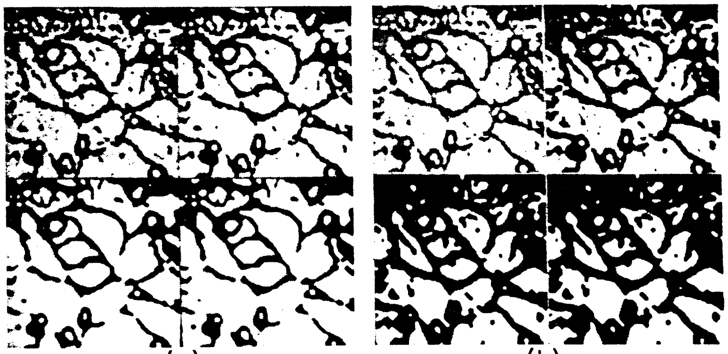
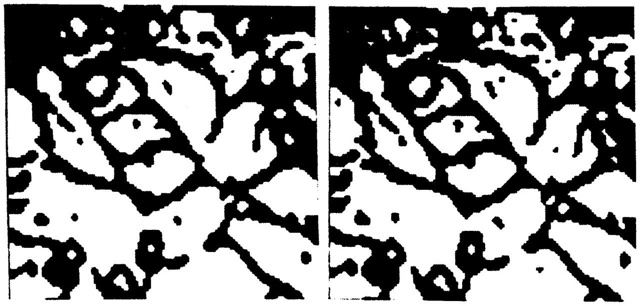


Fig. 3 Variation of Criterion (1) with the Iteration α_1/α_2 = FACT = 1



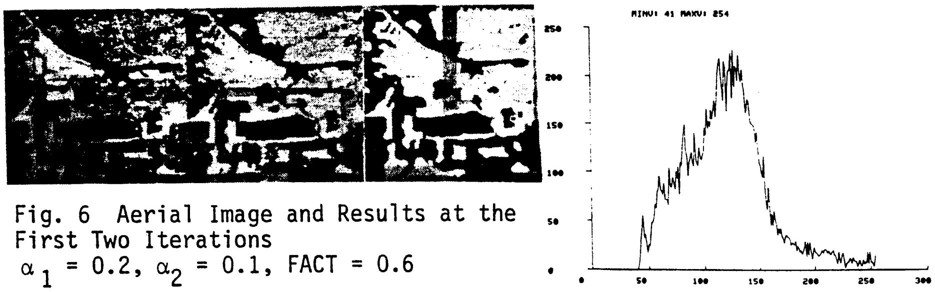
(a) $\alpha_1 = 0.4$, $\alpha_2 = 0.2$ (b) $\alpha_1 = 0.2$, $\alpha_2 = 0.4$

Fig. 4 Biasing of a Class. FACT = 1 in Both Cases



(a) $\alpha_1 = \alpha_2 = 0.2$, Iter.8 (b) $\alpha_1 = \alpha_2 = 0.9$, Iter. 3

Fig. 5 Illustration of the Speed of Convergence



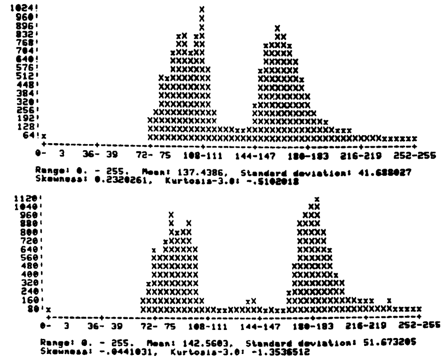
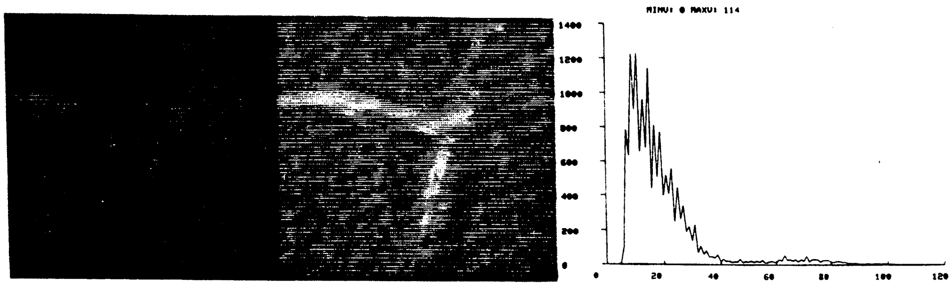


Fig. 6 Aerial Image and Results at the First Two Iterations
$\alpha_1 = 0.2$, $\alpha_2 = 0.1$, FACT = 0.6



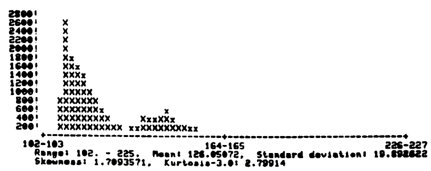
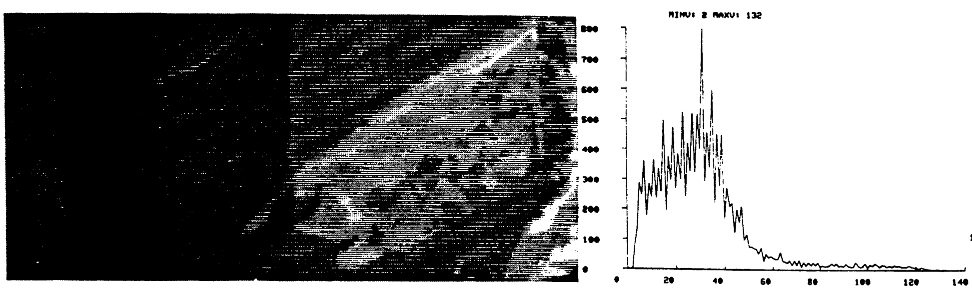


Fig. 7 Aerial Image and Results at the First Iteration
$\alpha_1 = 0.1$, $\alpha_2 = 0.05$, FACT = 1



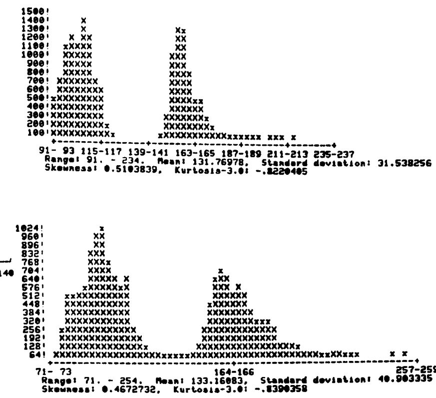
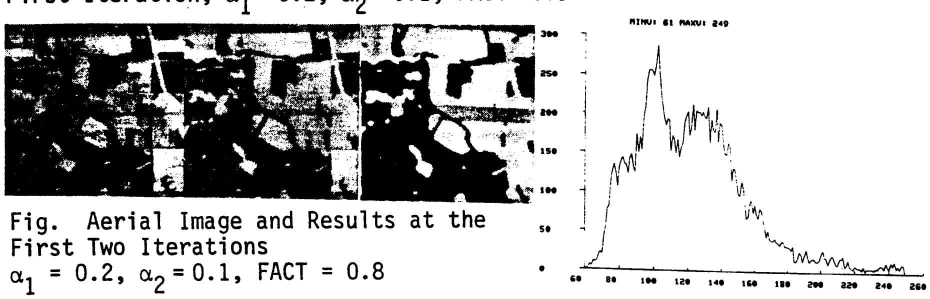


Fig. 8 Aerial Image and Results at the First Iteration, $\alpha_1 = 0.2$, $\alpha_2 = 0.1$, FACT = 0.9



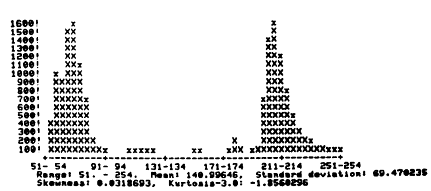


Fig. Aerial Image and Results at the First Two Iterations
$\alpha_1 = 0.2$, $\alpha_2 = 0.1$, FACT = 0.8

IEEE Catalog No. 82CH1801-0
Library of Congress No. [illegible]
Computer Society Order No. [illegible]

VOLUME
2

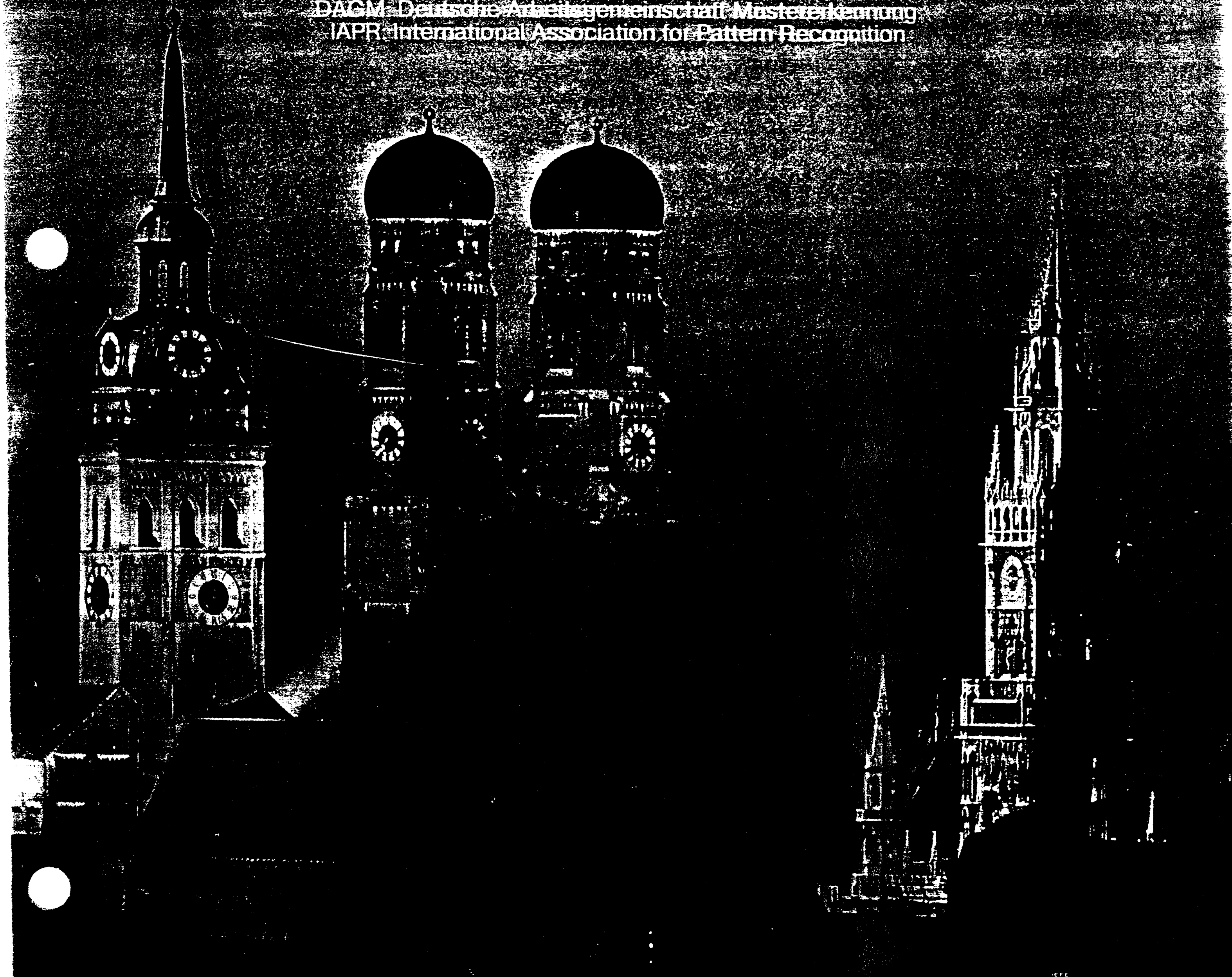
PROCEEDINGS

6th International Conference on

PATTERN RECOGNITION

Munich, Germany, Oct. 19-22, 1982

Organizers
DAGM: Deutsche Arbeitsgemeinschaft Mustererkennung
IAPR: International Association for Pattern Recognition



IEEE COMPUTER SOCIETY   THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, INC.   IEEE COMPUTER SOCIETY PRESS

PROCEEDINGS

PATTERN RECOGNITION

IEEE COMPUTER SOCIETY PRESS

IEEE Catalog No. 82CH1801-0
Library of Congress No. 82-82809
Computer Society Order No. 436

DAGM: Deutsche Arbeitsgemeinschaft Mustererkennung

IAPR: International Association for Pattern Recognition

IEEE COMPUTER SOCIETY

THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, INC.