Global-to-Local Non-Rigid Shape Registration

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Abstract

Non-rigid shape registration is an important issue in computer vision. In this paper we propose a novel global-to-local procedure for aligning non-rigid shapes. The global similarity transformation is obtained based on the corresponding pairs found by matching shape context descriptors. The local deformation is performed within an optimization formulation, in which the bending energy of thin plate spline transformation is incorporated as a regularization term to keep the structure of the model shape preserved under the shape deformation. The optimization procedure drives the initial global registration towards the target shape that results in the one-to-one correspondence between the model and target shape. Experimental results demonstrate the effectiveness of the proposed approach.

1. Introduction

Registration of non-rigid shapes is an important issue in computer vision and it has drawn an increasing attention due to its wide range of application related to recognition, tracking and retrieval. The shape registration problem can be stated as follows: given two shapes, a model shape M and a target shape S, find the best transformation that assigns any point of M a corresponding point in S and minimizes the dissimilarity between the transformed shape $\hat{M}$ and S. Therefore, there are two problems to be resolved: the correspondence, and the transformation.

The non-rigid shape registration is much harder since it has more degrees of freedom than the rigid shape registration problem. Recently researchers have come up with different approaches to solve the non-rigid shape registration problem [1, 3, 4, 7, 9]. Chui and Rangarajan [3] presented an optimization based approach, TPS-RPM (thin plate spline-robust point matching) algorithm, to jointly estimate the correspondence and non-rigid transformations between two point-based shapes. Belongie et al. [1] proposed a descriptor called *shape context* to find correspondences by minimizing the overall shape context distances and the TPS transformation is iteratively solved. Guo et al. [4] described a joint clustering and diffeomorphism estimation algorithm that can simultaneously estimate the correspondence and fit the diffeomorphism between the two point sets (a diffeomorphism is a invertible function that maps one differentiable manifold to another such that both the function and its inverse are smooth). Paragios et al. [7] introduced a simple and robust shape representation (distance functions) and a variational framework for global-to-local registration in which a linear motion model and a local deformation field are incrementally recovered. Zheng and Doermann [9] presented a relaxation labeling based point matching algorithm for aligning non-rigid shapes. The point matching is formulated as a graph matching problem to preserve local neighborhood structures in which the point is a node and neighboring points are connected by edges.

As compared to these approaches, we decompose the non-rigid shape registration into a two-step procedure: the global similarity transformation that brings the model shape and target shape into a coarse alignment and the local deformation that deforms the transformed model shape to the target shape. For the first step, feature based registration is employed; in the second step the local deformation is formulated as an optimization problem to preserve the structure of the shape model.

Contributions of this paper: The contributions of this paper are: (a) A novel global-to-local procedure for aligning non-rigid shapes is introduced. (b) The bending energy of TPS is incorporated into an optimization formulation as a regularization term to preserve the structure of the model shape under the shape deformation.

2. Technical Approach

2.1 Global Similarity Registration

The task of the global registration is to find a similarity transformation between M and S that includes three parameters (s, θ, T); a scale factor s, a rotation θ and a translation vector $T = (T_x, T_y)$. Once the corresponding pairs (M_i, S_i) are known, the three parameters can be estimated by minimizing the sum of square distance between

The 18th International Conference on Pattern Recognition (ICPR'06)
0-7695-2521-0/06 $20.00 © 2006 IEEE

the transformed M_i and S_i.

In this paper, the shape context descriptor [1] is used to find the correspondence only. Considering the shape contexts of two point a and b, the cost C_{ab} of matching the two points is evaluated by χ^2 test statistic since shape contexts are histograms that can approximate probability distributions.

$$C_{ab} = C(a,b) = \frac{1}{2}\sum_{i=1}^{K}\frac{(h_a(i)-h_b(i))^2}{h_a(i)+h_b(i)} \tag{1}$$

where $h_a(i)$ and $h_b(i)$ denote the $K-$bin normalized histograms at points a and b respectively. Given the sets of costs $C(a,b)$ between pairs a on the model shape and b on the target shape, the optimal correspondence is found by minimizing the sum of individual matching costs. This is solved by a bipartite matching algorithm with the one-to-one matching constraint [1]. In our case, the configuration of the model shape is fixed and we resample the points from the contour of the model shape. Then we compute shape context descriptors to find the correspondences of the model shape in the target shape to calculate the global similarity transformation, while in [1] the non-linear TPS transformation is solved iteratively by warping the model shape to the target and recovering correspondences.

2.2 Thin Plate Spline Transformation

Since the global registration can not account for the local deformation, the transformed model shape needs to deform to the target shape after the global alignment. Thin plate spline (TPS) transformation is a powerful tool for modeling the shape deformation and is widely used in shape matching [2, 3, 6, 8]. TPS defines a mapping from $\mathcal{R}^d$ to $\mathcal{R}$ where d is the dimension. The TPS $\mathcal{R}^2 \rightarrow \mathcal{R}^2$ mapping function is defined by the following equation:

$$\mathbf{v} = f(\mathbf{u}) = \begin{bmatrix} f^x(\mathbf{u}) \\ f^y(\mathbf{u}) \end{bmatrix} = A\mathbf{u} + \mathbf{t} + \sum_{i=1}^{n}\begin{bmatrix} w_i^x \\ w_i^y \end{bmatrix}\phi(|\mathbf{u}-\mathbf{u}_i|) \tag{2}$$

where $\phi(r) = r^2 log r$, $\mathbf{u} = [\hat{x},\hat{y}]^T$, $\mathbf{v} = [x,y]^T$ and A and $\mathbf{t}$ form an affine transformation given by $[A \quad \mathbf{t}] = \begin{bmatrix} a_{00} & a_{01} & t_0 \\ a_{10} & a_{11} & t_1 \end{bmatrix}$. The $n \times 2$ matrix $W = \begin{bmatrix} w_1^x & w_2^x & \cdots & w_n^x \\ w_1^y & w_2^y & \cdots & w_n^y \end{bmatrix}^T$ specifies the non-linear warping and n is the number of control points. Given n control points $\mathbf{u}(\hat{x}_i,\hat{y}_i)$ and their corresponding points $\mathbf{v}(x_i,y_i)$, equation (2) can be rewritten as $2n$ linear equations. However there are $2n+6$ ($2n$ non-linear warping parameters W and 6 affine parameters $[A \quad t]$) unknown parameters to be solved, the following six constraints are added to make the spline function (equation (2)) have square integrable second derivatives:

$$P^T[w_1^x, w_2^x, \cdots, w_n^x]^T = 0, \;\; P^T[w_1^y, w_2^y, \cdots, w_n^y]^T = 0 \tag{3}$$

where P is a $n \times 3$ matrix which is defined by $(\mathbf{1}, \hat{\mathbf{x}}, \hat{\mathbf{y}})$, $\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n)^T$ and $\hat{\mathbf{y}} = (\hat{y}_1, \hat{y}_2, \cdots, \hat{y}_n)^T$. The $2n+6$ equations can be put into a compact matrix form:

$$\begin{bmatrix} \Phi & P \\ P^T & 0 \end{bmatrix}\begin{bmatrix} W \\ \mathbf{t}^T \\ A^T \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{0} \end{bmatrix} \tag{4}$$

where the $n \times n$ matrix $\Phi_{ij} = \phi(\mathbf{u}_i - \mathbf{u}_j)$, $\mathbf{v} = (\mathbf{x}, \mathbf{y})$, $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T$ and $\mathbf{y} = (y_1, y_2, \cdots, y_n)^T$. The TPS transformation minimizes the following bending energy function,

$$B_e = \iint_{\mathcal{R}^2}(F(f^x) + F(f^y))dxdy \tag{5}$$

where $F(g) = (g_{xx}^2 + 2g_{xy}^2 + g_{yy}^2)$. It can be shown that the value of bending energy is $B_e = \frac{1}{8\pi}(\mathbf{x}^T K \mathbf{x} + \mathbf{y}^T K \mathbf{y})$ where $\mathbf{x} = (x_1, x_2, \cdots, x_n)^T$ and $\mathbf{y} = (y_1, y_2, \cdots, y_n)^T$ [2]. The matrix K is the $n \times n$ upper left matrix of $\begin{bmatrix} \Phi & P \\ P^T & 0 \end{bmatrix}^{-1}$, which only depends on the configuration of the control points in $\{\mathbf{u}\}$. Therefore, the bending energy is determined by the configuration of control points and their correspondences. The bending energy is a good measurement of the shape deformation. For non-rigid shape registration, the model shape is fixed, which means the matrix K is fixed and can be precomputed. Since our task is to drive the model shape towards the target shape with the structure of the model shape preserved, the bending energy can be used to penalize the large deformation.

2.3 Optimization Formulation

After the model shape is brought into coarse alignment with the target shape through the global registration, it needs to deform to the target shape. In other words, we like to drive the model shape more close to the target shape with the structure of the shape model preserved. We can achieve this task by minimizing the proposed cost function:

$$\begin{aligned} E(\mathbf{x},\mathbf{y}) &= E_{img}(\mathbf{x},\mathbf{y}) + \gamma E_D(\mathbf{x},\mathbf{y}) \\ &= \sum_{i=1}^{n} g(|\nabla I_m(x_i,y_i)|) + \frac{1}{2}\gamma(\mathbf{x}^T K \mathbf{x} + \mathbf{y}^T K \mathbf{y}) \end{aligned} \tag{6}$$

where $g(|\nabla I_m|) = 1/(1 + |\nabla I_m|)$, $|\nabla I_m(x_i,y_i)|$ is the gradient magnitude of ith point of the model shape located in the image I_m and γ is a positive regularization constant that controls the structure of the shape model. For example, increasing the magnitude of γ tends to keep the structure of the shape model unchanged. In equation (6) the first term E_{img} drives points $(\mathbf{x},\mathbf{y})$ towards the target shape; the second term E_D is the bending energy that preserves the structure of the model shape under the shape deformation. When we take the partial derivatives of equation (6) with respect to $\mathbf{x}$ and $\mathbf{y}$ and set them to zero, we have

$$\gamma K\mathbf{x} - \sum_{i=1}^{n} \frac{1}{(1+|\nabla I_m(x_i,y_i)|)^2} \frac{\partial |\nabla I_m(x_i,y_i)|}{\partial \mathbf{x}} = 0,$$
$$\gamma K\mathbf{y} - \sum_{i=1}^{n} \frac{1}{(1+|\nabla I_m(x_i,y_i)|)^2} \frac{\partial |\nabla I_m(x_i,y_i)|}{\partial \mathbf{y}} = 0. \quad (7)$$

Since K is positive semidefinite, equation (7) can be solved iteratively by introducing a step size parameter α which is shown in equation (8) [5].

$$\gamma K\mathbf{x}_t + \alpha(\mathbf{x}_t - \mathbf{x}_{t-1}) - \mathcal{F}_x\Big|_{\mathbf{x}=\mathbf{x_{t-1}},\mathbf{y}=\mathbf{y_{t-1}}} = 0$$
$$\gamma K\mathbf{y}_t + \alpha(\mathbf{y}_t - \mathbf{y}_{t-1}) - \mathcal{F}_y\Big|_{\mathbf{y}=\mathbf{x_{t-1}},\mathbf{y}=\mathbf{y_{t-1}}} = 0 \quad (8)$$

The solution is obtained by iteratively solving equation (9) where I is the identity matrix until the cost function $E(\mathbf{x},\mathbf{y})$ is not reduced. In equation (8) and (9), $\mathcal{F}_x = \sum_{i=1}^{n} \frac{1}{(1+|\nabla I_m(x_i,y_i)|)^2} \frac{\partial |\nabla I_m(x_i,y_i)|}{\partial \mathbf{x}}$ and $\mathcal{F}_y = \sum_{i=1}^{n} \frac{1}{(1+|\nabla I_m(x_i,y_i)|)^2} \frac{\partial |\nabla I_m(x_i,y_i)|}{\partial \mathbf{y}}$.

$$\mathbf{x}_t = (\gamma K + \alpha I)^{-1}\Big(\alpha \mathbf{x}_{t-1} + \mathcal{F}_x\Big|_{\mathbf{x}=\mathbf{x_{t-1}},\mathbf{y}=\mathbf{y_{t-1}}}\Big)$$
$$\mathbf{y}_t = (\gamma K + \alpha I)^{-1}\Big(\alpha \mathbf{y}_{t-1} + \mathcal{F}_y\Big|_{\mathbf{x}=\mathbf{x_{t-1}},\mathbf{y}=\mathbf{y_{t-1}}}\Big) \quad (9)$$

3. Experimental Results

We used the shape database collected by Brown university [1]. Figure 1 shows the non-rigid shape global-to-local registration results on three shapes of the hand, dude and fish. Figure 1(a) shows three different model shapes; Figure 1(b) shows the model shape and target shape before the global registration; Figure 1(c) shows the transformed model shape overlaid on the target shape after the global alignment; Figure 1(d) shows the results after local shape deformation; Figure 1(e) shows the one-to-one correspondence which is connected in blue lines; Figure 1(f) shows warped regular grids by the local deformation computed from equation 4. From Figure 1, we see that the model shape is successfully deformed to the target shape and the one-to-one correspondence is established. Figure 2 shows an example of deforming the model shape (dude) to the target shape at iteration 0, 25, 50, 75, 100 and 125. We observe that the model shape is driven towards the target shape by the optimization formulation. The regularization term γ for shapes of the hand, dude and fish is 20, 100, 25 respectively.

Quantitative evaluation: The accuracy of the proposed approach is quantified by the average Euclidean distance between the points in the transformed model shape and the correspondences in the target shape. The average distances after the local deformation for the shapes of hand, dude and fish are 0.59, 0.45 and 0.61 pixels respectively; while the distances after global alignment are 1.84, 1.80 and 2.49 pixels. We observe that the proposed global-to-local non-rigid shape registration algorithm brings the model shape and the target shape into good alignment.

[1] http://www.lems.brown.edu/vision/

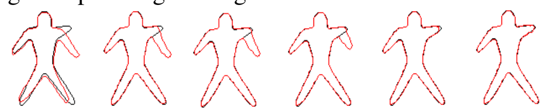

Figure 2. Deformation of the transformed model shape to the target shape (The model shape in red, the target shape in black).

4. Conclusions

In this paper, we have proposed a novel global-to-local procedure for aligning non-rigid shapes. Since the structure of the model shape should be preserved under the deformation, the bending energy is incorporated into the optimization formulation as a regularization term to penalize the large shape deformation. The optimization procedure drives the initial global registration towards the target shape with the structure of the model shape preserved and finally finds the one-to-one correspondence between the model shape and target shape. Experimental results on three non-rigid shapes show the effectiveness of the proposed approach.

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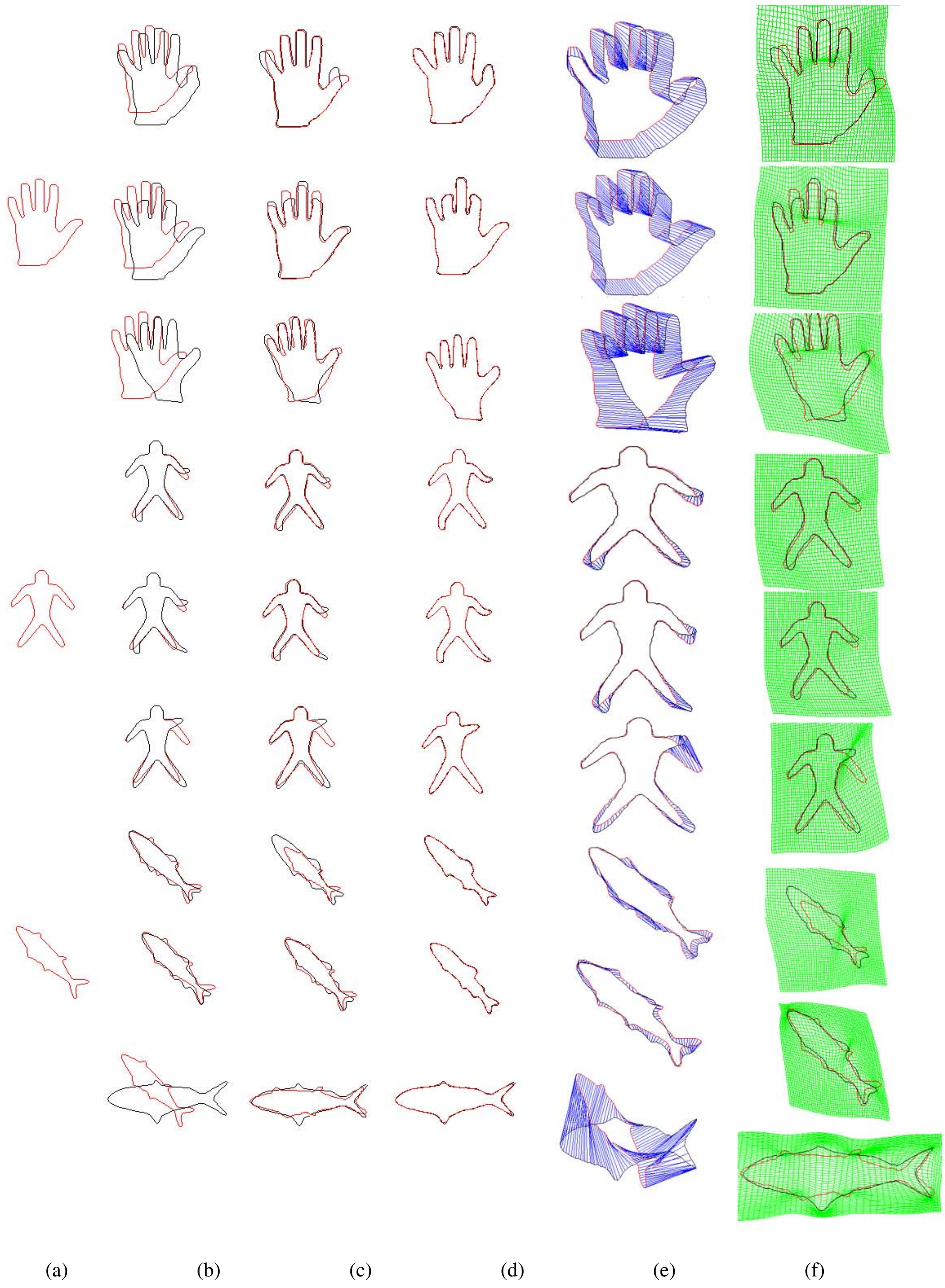

Figure 1. Global-to-local registration results on three shapes of the hand, dude and fish (The model shape in red, the target shape in black). (a) Model Shape. (b) Initial condition. (c) Global registration. (d) Local deformation. (e) Correspondences. (f) Grid deformation.